Improved arrangement of shock-detecting pins in shock equation of state experiments

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Shock wave speeds are often measured by comparing arrival times at the tips of electrical shorting pins in a hexagonal array over two elevations (called up and down). In the conventional arrangement, the center pin is solely responsible for measuring the curvature of the wave front. Without this datum the shock speed cannot be precisely determined. In some experiments this pin fails frequently enough to be a problem. We report a simple rearrangement between up and down designated pins which eliminates the critical reliance on a single pin. © *1995 American Institute of Physics.*

I. INTRODUCTION

An important equation of state measurement is the velocity of shock waves generated in a sample by impact from a high velocity (several km/s) disk projectile. Such experiments[1–5] are vitally important in physics for establishing to high precision the behavior of materials at high pressure, including materials used by researchers as standards. In many cases, a shock wave experiment is the only method available to attain the necessary pressure, temperature, or to perform an accurate measurement over a sufficiently large sample volume. The shock velocity experiment is the preferred method to determine the equation of state of standards because the developed shock parameters depend through simple relations[3] on only four values which can potentially be measured very precisely: projectile velocity and initial density, initial sample density, and the measured shock velocity (U_s). Experimental techniques have been developed[1,2] to accurately measure these parameters using a two-stage gas gun.[6] These account for the bowing distortion the projectile suffers from the acceleration of the launch. In experiments involving thin samples and high projectile velocities the shock transit times are short; thus, it is increasingly important to precisely account for the projectile distortion. We report on a simple modification in the target design which will significantly reduce the uncertainty in the shock speed determination.

II. TARGET DESIGN

Figure 1 is a schematic of the target used to measure shock speeds. A metal disk projectile 1–3 mm thick is accelerated up to 8 km/s by a two-stage gas gun.[1,6] Impact with the sample creates a shock wave propagating toward the rear of the sample. Passage of the shock through specific locations around the sample is observed by a set of electrical shorting pins. They are placed in two planes (elevations) parallel to the face, called "up" and "down," which are separated by a distance S (the step height). For samples of 15–20 mm diameter, S is 1–3 mm.

The pins generate an electrical pulse upon passage of the shock which is sent via cable to the recording electronics. These shock arrival times at the target can be determined to ~0.5 ns overall precision,[1] including cable length uncertainties. (At 10 km/s, this time precision corresponds to measuring pin elevations to a precision of 5 μm.) In the measurement of shock speed in a stiff material such as sapphire,[7] the shock transit time can be as low as 100 ns. Thus, this subnanosecond time precision is important.

There is an uncertainty in the shock arrival time pulse generated by a pin, due to variations in the thickness or shape of the tip, or the presence of foreign matter under the tip, all which change the effective elevation. The pins must be pressed securely against the sample by springs to ensure their location at the measured elevation surface. In some pin types, vibration from handling the target can rub off the metal film at the tip, causing the pin to fail. For these reasons, targets have as many pins as possible for redundancy. Due to the width of the pins and springs, it is difficult to space pins closer together than ~3 mm center to center. Avoidance of the release waves from sample corners reduces the available area on the up elevation. Previous[5,6] researchers using larger targets have used pins side by side in a line. For our smaller samples this is not practical. Instead, a hexagonal pin arrangement provides the most efficient packing.

Figure 2 diagrams the arrangement of the pins. Each pin is either on the up or down elevation. There are two configurations, labeled "snowflake" and "pinwheel." Both use thirteen pins divided into six down, six up, and one center pin. The snowflake pattern is the conventional configuration used in some previous equation of state studies.[1–3] Here, the center pin is crucial for determining the degree of radial distortion called bowing. We will show below that without this pin the transit time cannot be precisely determined. Furthermore, even with valid data from this pin the precision for the entire transit time determination is limited by the uncertainty of this single pin, not benefitting from averaging over the twelve other pins.

The latter configuration (pinwheel) is introduced by this report as a solution to this critical dependence on the center pin. We will show that with the pinwheel arrangement the center pin is not needed to accurately determine the shock transit time.

III. DISTORTION MODES

For a steady shock, the shock front surface retains its shape as it propagates through the sample.[5] This surface is

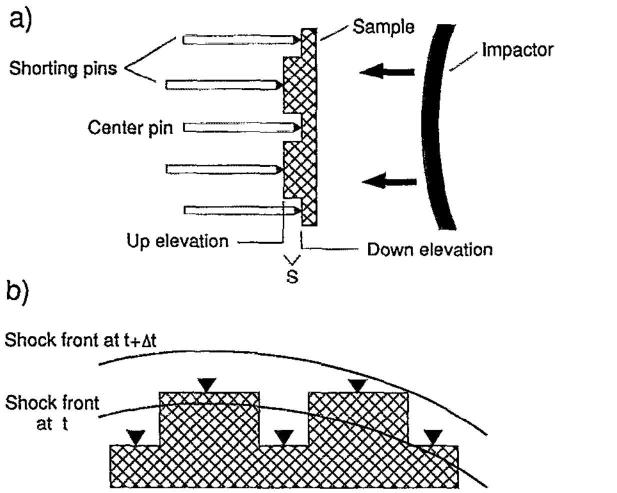

FIG. 1. The target for measuring shock speed, side view. (a) The sample is a "tophat" having elevations at two levels up and down differing by the height S. Other supporting structures are not shown. The impactor is a metal disk, which generally has a bow and tilt due to acceleration of the launch. Electrical shorting pins pressed against elevation surfaces detect the arrival of the shock wave. (b) Details of the shock front in relation to the pin tips (triangles). The vertical dimension is exaggerated. The shock front preserves its shape as it moves perpendicular to the target face. The shock front is shown at two moments separated by time interval Δt.

determined by the tilt and shape of the projectile at impact. Figure 3 shows the three most significant kinds of distortion, presented in order of the typical magnitude observed in ten shots with a 1-mm-thick Ta projectile at speeds from 4 to 8 km/s. The most prevalent distortion mode is tilt [Fig. 3(a)]. Although not colloquially termed a distortion, mathematically it is the lowest angular mode of deviation from a perpendicular plane, varying as sin θ, where θ is an angle measured from the center. In contradistinction to the other distortion modes, its magnitude typically decreases for increasing projectile velocity. For 4–8 km/s it ranged from 20 to 50 ns measured across the diameter $2R_b \approx 20$ mm.

Figure 3(b) shows the lowest radial mode called bowing, which has a parabolic character. This is the most important distortion relevant to our discussion comparing the snowflake and pinwheel pin arrangements. Typical deviations measured from the center to the perimeter ranged from $+7$ to -16 ns. (Positive bowing is a concave projectile). Mitchell

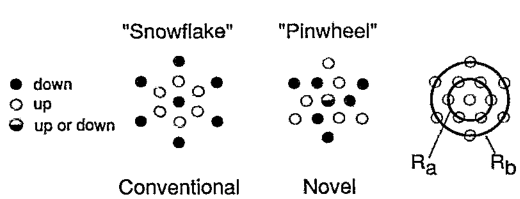

FIG. 2. Arrangement of the up and down designated electrical shorting pins on the sample, projected on the target face. The inner six pins are on a diameter of radius R_a, the outer six on R_b. Both the snowflake and pinwheel arrangements have thirteen pins, split seven/six between down and up elevations. However, the pinwheel has pins of both kinds on each radius. The center pin in the pinwheel can be up or down. The snowflake center pin should be of the same kind as the outer pins to best resolve the bowing. The up/down roles as a group can be interchanged.

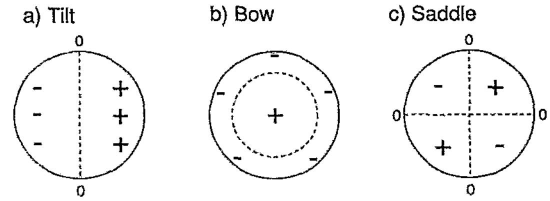

FIG. 3. A map of the impactor distortion modes, in order of typical magnitude. The plus and minus signs indicate deviation from the perpendicular plane. The dashed line or curve is a locus of zero deviation. (a) Tilt, typically 20–50 ns across the face. (b) Spherical bow of either polarity, typically up to 15 ns, measured from center to periphery. (c) Saddle or potato-chip warp, typically 1 ns amplitude at the perimeter.

et al. observed[1,2] that the bowing dependence on impactor thickness and velocity was irregular.

A much smaller distortion is a "saddle" or "potato chip" mode [Fig. 3(c)], which is the second angular mode. The deviation from the plane perpendicular on the perimeter varies as sin 2θ. The typical amplitude at radius R_b was 0.5–1 ns.

Higher order angular and radial distortions have less than 1 ns amplitude. Mitchell *et al.* measured[1] the distortion of flyers using nineteen pins on three diameters and one center pin. The additional diameter allowed resolution of nonparabolic radial distortion. Their data indicated that any higher radial mode had less than ~ 1 ns amplitude. Higher angular modes must have amplitudes <1 ns, because this is the standard deviation of fits to the saddle mode in our observations. The argument now being made concerning the critical dependence of the center pin does not depend on the detailed structure of the modes or the net distortion, only on the observation that the bowing component is an order of magnitude more prevalent than the saddle component and not vice versa.

IV. FINDING THE TRANSIT TIME

Figure 4 illustrates the data analysis method for any pin arrangement. The shock arrival times are plotted versus the two-dimensional pin location projected across the target face. If the actual vertical location of a pin deviates slightly from the theoretical elevation plane, this vertical difference is converted into a time difference by the anticipated shock velocity and the arrival time datum is adjusted accordingly. The set of the up pin data is shifted vertically as a group by an adjustable parameter Δt until it interpenetrates the down data to form a best-fit surface. The resulting value of Δt is the shock transit time across the step height S and yields the shock speed $U_S = S/\Delta t$. This interpenetration method has been previously employed in one dimension.[8]

A few words should be said about the definition of the best-fit surface. Several strategies are available. Since bowing and tilt are the dominant distortions, we could require the best-fit surface to be a paraboloid (i.e., preclude the saddle mode). This approach is mathematically related to the method used in Ref. 1, which involves fitting the arrival time data of a specific circle of pins (R_a and R_b) to a (A_i sin $\theta + B_i$) dependence, and then subtracting B_i values.

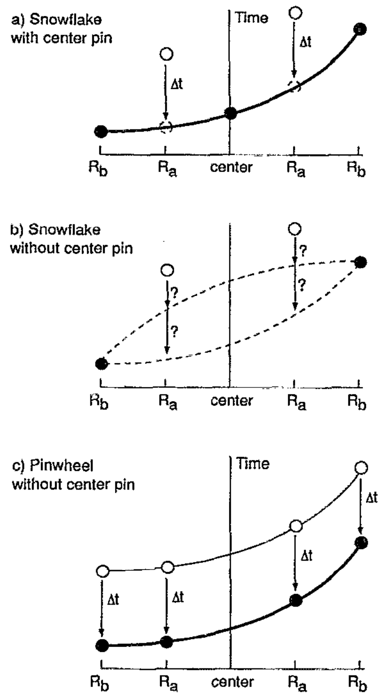

FIG. 4. Finding the transit time through interpenetration of data. Arrival times of shocks at pins are plotted vs two-dimensional pin location across the face. Here only one dimension is shown on the horizontal axis. The open and closed circles represent up and down pin data, respectively. Here, the center pin is a member of the down set. The up data are translated in time by Δt to form a mutual best-fit surface. This yields the shock speed $U_S = S/\Delta t$, where S is step height. (a) Pins in a snowflake arrangement. (b) If the center pin is missing in the snowflake arrangement, Δt cannot be determined because curvature of the shock surface is indeterminate. Fundamentally, this is because R_a and R_b contain only up and down pins, respectively. (c) In the pinwheel arrangement, the lack of a center pin does not prevent accurate determination of Δt because the circles R_a and R_b each contain both up and down pins.

However, the interpenetrating version of this procedure is more general since it does not require the pin locations to lie on a circle or in any predetermined pattern.

An alternative strategy does not presume the mathematical class of the best-fit surface *a priori*. Instead, it requires that the "surface energy" of the best-fit surface be minimized. This is as if one had two interpenetrating wire frames covered with a soap film, and allows the frames to come to equilibrium—thereby minimizing the soap film surface area. This method has the advantage of not precluding any distortion modes. Second, to the extent that the projectile disk surface in flight may be controlled by an analogous minimum surface area process, this algorithm may more closely model realistic distortions without explicitly resolving them into radial and angular harmonics. Third, the calculation of the surface area is easily implemented by computer.

V. CRITICAL DEPENDENCE ON CENTER PIN

The discussion of best surface definition is a digression from our principle point: without the center pin the bowing cannot be determined using the snowflake pin arrangement [Fig. 4(b)]. Even when the center pin datum is valid, the precision of Δt is limited by the uncertainty (δt_c) of the center pin time t_c. For a demonstration of this, we will analyze the origin of Δt using an analysis similar to Ref. 2. Figure 5 illustrates our variable definitions. The centers of best-fit planes through sets of up and down pins define t_u and t_d, respectively. For simplicity, let $R_a = R_b/2$. Let the bow amount (Δt_{bow}) be measured between t_c and t_d. For a paraboloidal surface the amount of bow at R_a is 75% of Δt_{bow}. The transit time Δt is equal to

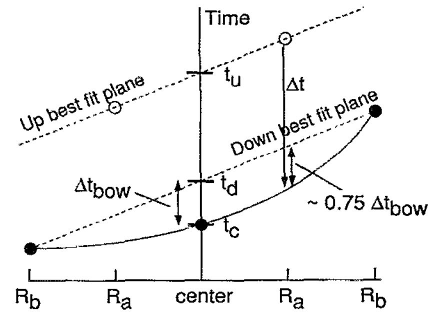

FIG. 5. Variable definitions in discussing the importance of the center pin. The open/closed circles are the up/down pins. The centers of the best-fit planes through up and down pins define t_u and t_d. The difference between t_d and center pin arrival time t_c is the bow amount Δt_{bow}. Since $R_a \approx 0.5\ R_b$ and the bow is parabolic, the transit time is $\Delta t \approx t_u - 0.25 t_d - 0.75 t_c$. Thus, even if t_d were known infinitely precisely, the uncertainty in Δt is not smaller than 75% of the uncertainty in the center pin time t_c.

$$\Delta t = (t_u - t_d) + 0.75 \Delta t_{\text{bow}} = t_u - 0.25 t_d - 0.75 t_c. \quad (1)$$

Thus, even if t_d and t_u were known infinitely precisely, the uncertainty of the final measurement[9] would be at least as great as $\sim 0.75\ \delta t_c$. This is a disappointment, since we intuitively expect that by using $2N$ pins to make the transit time determination that the uncertainty would be $\sim \sqrt{2}\,\delta t/\sqrt{N}$, where δt characterizes the uncertainty of a typical pin. Thus, the benefit of multiple pins to decrease the net uncertainty is not achieved in the snowflake arrangement.

Fundamentally, this is because in the snowflake arrangement on a given circle (R_a or R_b), all the pins are the same kind, up or down. In the pinwheel arrangement, there are both up and down pins on the same circle, and thus the center pin is not required to determine Δt [Fig. 4(c)].

To show an extreme example of the robustness of the pinwheel pattern, suppose we lose all pins except for three up and three down pins all on one circle R_a or R_b. Then interpenetrating the up and down pins so they all lie on a circle will give us Δt accurate to 1 ns. (The latter is the typical maximum amplitude of the saddle distortion, which cannot be separated from Δt using only three pins.)

A more likely scenario is that only one or two pins fail. In these cases Δt will be more accurately determined, and both the bowing and saddle components will be separated from Δt. We expect the uncertainty of Δt in the pinwheel case to be of the order $\sim \sqrt{2}\,\delta t/\sqrt{N}$, in contrast to $\sim \sqrt{\delta t^2/N + 0.75^2 \delta t_c^2}$ (Ref. 9) of the snowflake. The former estimate is appropriate for a combination of $N+N$ pins fitting two best-fit surfaces with standard deviation δt.

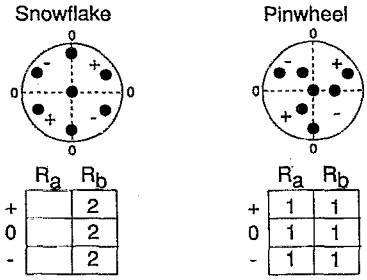

FIG. 6. Sensitivity to saddle mode distortion. A saddle mode deviation map is overlaid on the down pin set. Pins are at either the small (R_a) or large radius (R_b) from the center. The snowflake arrangement has two R_b pins sensing each of the positive, negative, and neutral excursions of the saddle distortion mode. The pinwheel arrangement has one R_a and one R_b pin sensing the same areas. Since the deviation increases linearly with radius from the center, and since $R_a \approx 0.5\ R_b$, the pinwheel arrangement has ≈75% of the sensitivity of the snowflake arrangement for the down pins. However, for the up pins the converse is true. The combined result is that the two arrangements have similar sensitivities.

VI. SENSITIVITY TO SADDLE MODE DISTORTION

One might initially think that the pinwheel would be less sensitive to saddle distortion than the snowflake. This because in the pinwheel, pins of one kind are arranged in what appears to be three arms, rather than six locations as in the snowflake. Thus, one might suspect a lesser sensitivity to the saddle distortion because of the Nyquist principle from digital sampling theory: there must be at least one sample for every half-period of the signal. Since the saddle mode has four half-periods around the circle, we might suspect that the three arms of the pinwheel are insufficient to detect this mode. However, the analysis illustrated in Fig. 6 shows that the pinwheel has only slightly reduced sensitivity, especially when the results of the up and down pins are combined (as uncorrelated results).

Consider that the pinwheel samples the distortion at six places around a circle, albeit, three of those locations are at the smaller radius R_a instead of R_b. The net sensitivity is proportional to a sum over the magnitude of the distortions at each location. The angular mode distortion increases linearly with radius. Thus one essentially sums over the number of pins at the + or − deviation areas weighted by their radii—hereafter referred to as the *moment*.

Again, for simplicity use $R_a = 0.5 R_b$. Consider just the down pins, with the center pinwheel pin being down. Then the pinwheel moment is $(0.5+1+0.5+1)R_b = 3R_b$, and the snowflake moment is $(2+2)R_b = 4R_b$. Thus for the down pins, the snowflake is 33% more sensitive to the saddle mode. For the up pins, the pinwheel moment is the same $3R_b$, but for the snowflake it is $(2+2)R_a = 2R_b$. (These approximations neglect the slight effect of the lack of a center up pin.) Thus the up pin snowflake is 33% less sensitive than the pinwheel.

The up pin and down pin results are statistically uncorrelated, so we combine the uncertainties in quadrature. Suppose the individual uncertainties are reciprocally related to the moments. Then the uncertainty of the combined snowflake result is $\sim \sqrt{(4/3)^2 + (2/3)^2} = 1.49$ compared with the combined pinwheel result $\sim \sqrt{(1^2 + 1^2)} = 1.41$. Thus by this estimate, the pinwheel configuration does not have a significantly less overall sensitivity to the saddle distortion.

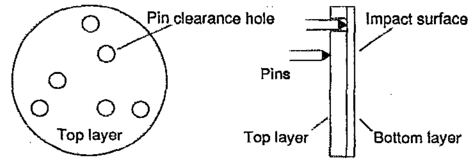

FIG. 7. The pinwheel target implementation using clearance holes.

VII. IMPLEMENTATION

Whereas the sample in the snowflake configured target can be machined on a lathe from a single piece, this is not possible with the pinwheel arrangement. However, the pinwheel (and snowflake) can be implemented from two layers of the sample, glued together after the upper layer has six clearance holes drilled in it. (Fig. 7). Since the interpin separation is approximately the same between snowflake and pinwheel configurations, the step height can be the same (when considering release waves from the corners).

VIII. DISCUSSION

We believe the pinwheel pattern is the most robust to pin loss, has the least uncertainty in transit time, and most space efficient arrangement of thirteen pins for shock wave transit time experiments. This is important in experiments where the degree of radial bowing is significant compared to the shock transit time. Such is the case in stiff materials having high shock and sound speeds since the transit times are short, the sample dimensions need to be small, and the projectile distortion significant and not reproducible from shot to shot. For example, in the 10 Ta impactor shots previously mentioned, which were made studying sapphire,[7] the center pin failed 30% of the time. We speculate, belatedly, that if those targets had the pinwheel pin configuration, we would not have had to discard three of those shots.

ACKNOWLEDGMENTS

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[9] The net uncertainty is analyzed in Ref. 2. However, their formulas Eqs. (26) and (28) are incorrect because they treat two correlated quantities [$\delta t_1, \delta t_c$, Eq. (6)] as uncorrelated. The correct expression for the net uncertainty at the single standard deviation level is $\delta(\Delta t)=[(R_a^2/R_b^2)^2\sigma_d^2/N_d+\sigma_u^2/N_u+\delta t_c^2(1-R_a^2/R_b^2)^2]^{1/2}$ where σ_d and σ_u are the standard deviations of the best-fit planes to the up and down pins, and N_u and N_d are the number of valid up and down pins, and δt_c is the uncertainty of the center pin.