Heterodyning Time Resolution Boosting for Velocimetry and Reflectivity Measurements

David J. Erskine

Lawrence Livermore Nat. Lab., 7000 East Ave, Livermore, CA 94550

ABSTRACT

A theoretical technique is described for boosting the temporal resolving power by several times, of detectors such as streak cameras in experiments that measure light reflected from or transmitted through a target, including velocity interferometer (VISAR) measurements. This is a means of effectively increasing the number of resolvable time bins in a streak camera record past the limit imposed by input slit width and blur on the output phosphor screen. The illumination intensity is modulated sinusoidally at a frequency similar to the limiting time response of the detector. A heterodyning effect beats the high frequency science signal down a lower frequency beat signal, which is recorded together with the conventional science signal. Using 3 separate illuminating channels having different phases, the beat term is separated algebraically from the conventional signal. By numerically reversing the heterodyning, and combining with the ordinary signal, the science signal can be reconstructed to better effective time resolution than the detector used alone. The effective time resolution can be approximately halved for a single modulation frequency, and further decreased inversely proportional to the number of independent modulation frequencies employed.

Keywords: velocity interferometry, Doppler, VISAR, heterodyning, streak cameras

1. INTRODUCTION

An important kind of measurement performed at national laboratories and some industrial research laboratories is the transit time of a shockwave or high pressure pulse passing through a thickness of material. This determines the speed of the wave, which in turn determines the material equation of state. Accurate equation of state data is needed to resolve computational material models which often differ only by a slight degree.

The shock arrival time is typically measured by a sudden change in sample reflectivity when illuminated by an essentially constant intensity (long laser pulse) and recorded by a streak camera. Measurements of the material velocity through the Doppler shift of monochromatic illumination are similarly performed, often in the same experiment, using a velocity interferometer whose phased outputs are recorded by a streak camera.

The recording device of choice is often a streak camera since this provides many parallel input channels, usually assigned to spatial location across a target. A significant limitation to the common streak camera is the maximum number of resolvable elements in the phosphor screen which presents the streak, and which sets the maximum number of resolvable time elements in the record, no matter what the sweep speed. Typically there are not more than 200 resolvable time bins, and will be less when used with increasing input slit width. Other streak camera defects include warping deviations of the electronic writing/streaking process which threatens accuracy.

The heterodyning technique described below effectively increases the number of resolvable time bins along the record, by decreasing the time resolution element while maintaining the same the record duration. It also makes the measurement more robust against warping deviations by embedding a network of timing fiducials with the science data everywhere in the record, not just along the side as conventionally done.

Further author information:
E-mail: erskine1@llnl.gov, Telephone: 1 925 422 9545

Alex11pp.tex 8/18/04

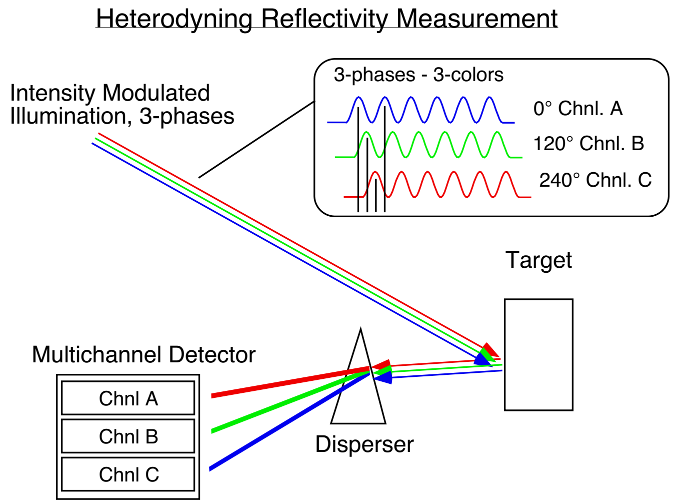

Figure 1. Heterodyning technique for measuring reflectivity at improved time resolution. This Figure shows only one point on the target, for simplicity. The illumination intensity is modulated sinusoidally at a frequency f_M. This causes reflectivity phenomena at frequency f to be heterodyned to lower frequency $(f - f_M)$, forming a beat signal recorded by the detector. The beat signal is restored to the original frequencies during analysis. To permit algebraic separation between the ordinary and beat components, which are embedded together in the recorded signal, the illumination is subdivided into 3 or 4 channels (a, b, c) have different phases separated by 120 or 90°. Here the phased channels are distinguished by wavelength. The channels can also be encoded by angle of incidence and/or polarization.

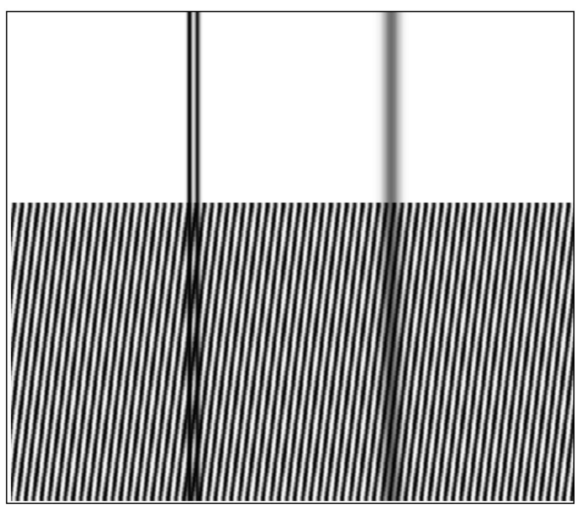

Figure 2. Graphical demonstration of the increased resolving power coming from a heterodyning effect. The classic definition of resolving power is ability to distinguish a doublet. If viewed from a distance, the doublet of lines on the left in the upper panel appears indistinguishable from the single line on the right. On the bottom panel a sinusoidal pattern is superimposed. The presence of the generated moiré pattern (beat signal) immediately distinguishes the doublet, hence the resolution has improved.

1.1. Heterodyning Measurement Technique

A heterodyning technique (Fig. 1) is described for improving the resolution and accuracy of temporal measurements, recorded by a multichannel detector (at least 3 channels). The technique can be employed for both reflectivity and velocity interferometer measurements. We will focus on reflectivity measurements, for concreteness. The apparatus differs from the ordinary reflectivity measurement by use of illumination $I(t)$ having periodic intensity modulation at a frequency f_m or period $T_m = 1/f_m$.

$$I(t) \propto [1 + \cos(2\pi t f_m)] \tag{1}$$

A science signal having high frequency components in the neighborhood of f_p is multiplied by the sinusoidal variation of $I(t)$ to form beats at a lower frequency $(f_p - f_m)$. These are more easily detected by the detector in spite of its limited frequency response ($f_D \approx 0.5/T_D$), where T_D is its response time.

The beat generation effect, or "heterodyning" can be illustrated graphically in Figures 2 and 3. Figure 2 uses the classical definition of resolving a pair of closely spaced lines. Note the appearance of a moiré pattern (beat signal) that easily distinguishes the doublet from the singlet. Figure 3 shows that a similar moiré pattern can be obtained from the edge of a step function, which is a better model for our shockwave application.

1.1.1. Phased Illumination

Let the illumination intensity for the n^{th} channel be

$$I_n(t) \propto [1 + \cos(2\pi t f_m + \phi_n)] \tag{2}$$

where ϕ_n is the illumination channel phase. The ϕ is ideally evenly distributed around the phase circle. For 3 channels ϕ is 0, 120 and 240°, and for 4 channels 0, 90, 180 and 270°. The independent channels could be encoded by wavelength, polarization, angle of incidence, spatial location on target (provided target has spatially uniform behavior), or some combination thereof. Figure 7 shows a method of encoding the channels using wavelength.

By measuring reflectivity data simultaneously in at least three separate channels where the illumination phase is different, it is possible to directly separate the ordinary and beat components arithmetically. At least three channels are needed to unambiguously determined phase and magnitude of the beat component.

Doppler velocimetry can also be performed by passing the reflected light from the target through an interferometer (often called a VISAR, for velocity interferometer system for any reflector[1]). The interferometer usually has multiple phased outputs, such as four in a push-pull system,[2] or even more when the interferometer phase is made to vary spatially across a line across the target, which is then measured by a streak camera. If the phased output of the VISAR is considered to be a complex signal, then the time varying magnitude is the sample reflectivity (times the illumination intensity) and the phase is proportional to the Doppler velocity.

1.1.2. Single-phased Illumination

An interesting aside is that single-phased illumination, not requiring multi-phase illumination, can also be used to perform heterodyning velocity interferometry. Having only a single channel of illumination is possible because the velocity interferometer provides the needed multiple phases (hence the multiphasing occurs after the target instead of prior). Having only one illumination channel greatly simplifies the hardware. However, the mathematics is not exactly analogous to the former case, and the separation of ordinary and beat components is not direct– but can be solved with iteration. Details of the single-phase illumination case will be left to a future article.

1.2. Benefits

1.2.1. Improved time resolution

Figure 4 shows the 1-d results of a numerical simulation measuring a perfectly sharp step, with and without heterodyning, using a single modulation frequency. The risetime has approximately doubled. Figure 5b shows the boosted frequency response possible using heterodyning at a single frequency modulation, and Fig. 6 with K multiple modulation frequencies. The temporal resolving power (proportional to the frequency response, and reciprocal of the time resolution element), is boosted by a factor $\sim (2K + 0.5)$.

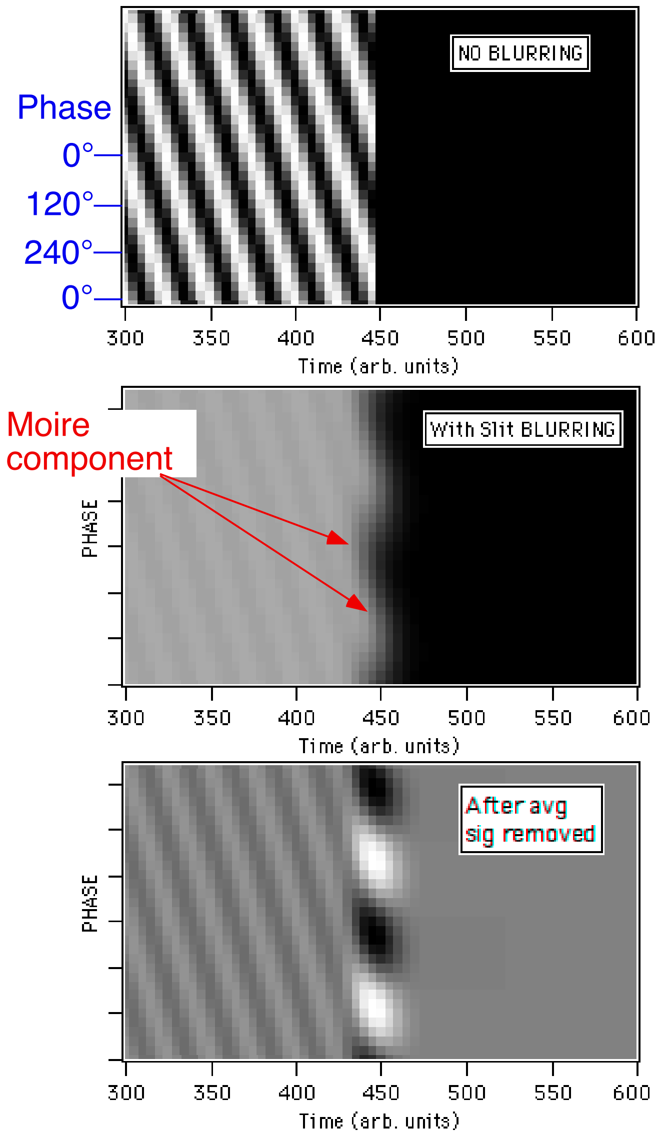

Figure 3. Graphical demonstration that a moiré (beat) signal can be generated from a step function. Simulated streak camera recording of a sudden change in reflectivity of a target illuminated by a periodically modulated illumination intensity. (a) Blurring is ignored. The modulation phase is plotted vertically so that all phases can be seen at once. (b) After detector blurring is imposed; note the wavy edge of the step, which is the moiré or beat component. (c) Removing the average signal highlights the moiré component. Its phase accurately measures the reflectivity edge location in spite of detector distortions, since the illumination and science signal distort together.

1.2.2. Robustness to streak sweep speed variations

The sinusoidal modulations are built-in time fiducial markers that allow the measurement to be independent from sweep speed variations and other display distortions, that often occur with streak cameras. The responsibility for time linearity now is removed from the detector and transferred to the illumination side of the apparatus, where it can potentially be more accurate.

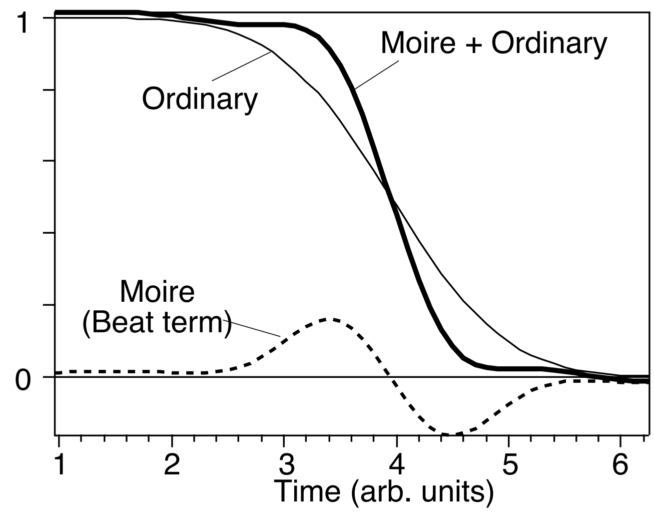

Figure 4. Numerically simulated results of conventional and heterodyning measurements, for the case of a perfectly sharp step function input signal and a detector having a Gaussian blur. The thin curve is the conventional result. The use of sinusoidal modulation creates a beat signal (dashed). This is added to the conventional signal to form a composite signal (bold) which has about twice as fast risetime.

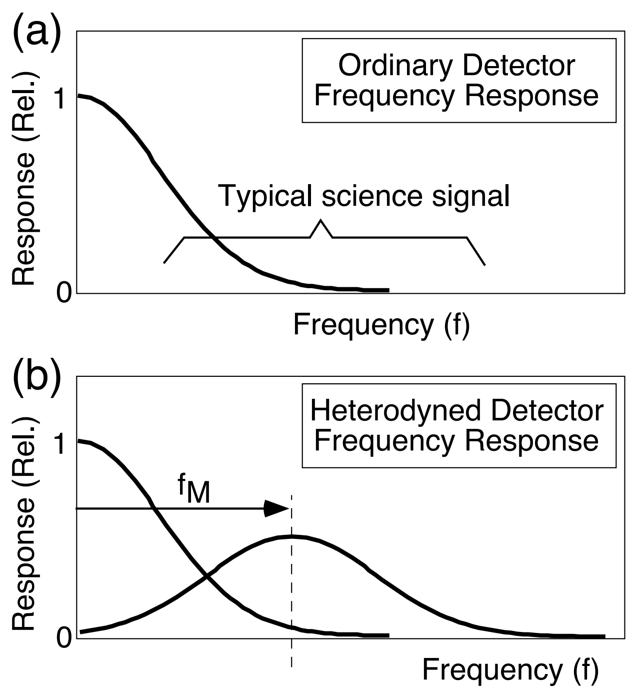

Figure 5. Idealized frequency response of the (a) conventional measurement and (b) heterodyning measurement. The most interesting portion of a typical science signal is often in the highest frequencies, which is where a detector is pushed to its performance limit. In the heterodyning technique (b) the illumination modulation creates a sideband in the response, which has the same shape as the ordinary response but shifted to higher frequency by f_M, (and one half the height). By choosing f_M to be at the shoulder of the ordinary response, the net frequency response is effectively extended.

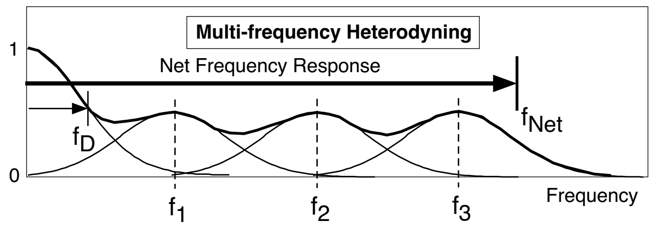

Figure 6. Frequency response when K multiple modulating frequencies (f_1, f_2, f_3) are employed in parallel. The net frequency response f_{net} is boosted over the conventional frequency response (f_D) by a factor $\Psi = f_{Net}/f_D \sim (2K + 0.5)$.

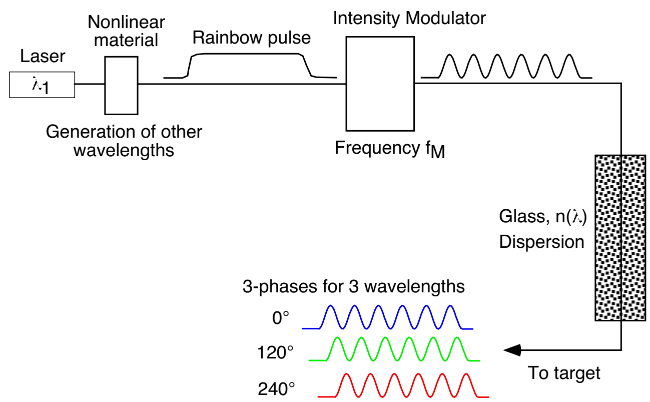

Figure 7. A method for making multi-phased illumination encoded by wavelength. An intense laser pulse passing through a nonlinear material generates a variety of wavelengths, all having similar duration. This broadband pulse passes through an intensity modulator where the same sinusoidal modulation at frequency f_M is imprinted on each. Passage through a dispersive medium such as glass delays shorter wavelengths relative to longer due to λ dependence of refractive index n. Glass length is chosen so that the delay (relative to $1/f_M$) between 3 distinguishable wavelengths is 120° of phase shift.

2. THEORY

2.1. Conventional detection

If $S(t)$ is the target reflectivity and $I(t)$ the illumination history, then the signal reaching the detector is $R(t) = S(t)I(t) = S(t)$, since for the conventional method $I(t) = 1$. The detected ordinary signal $B_{ord}(t)$, which is the blurred version of $R(t)$ and hence $S(t)$, is computed by a convolution with the detector impulse response $D(t)$,

$$B_{ord}(t) = S(t) \otimes D(t) \tag{3}$$

The convolution is more conveniently expressed as a multiplication in Fourier-space,

$$\mathbf{b}_{ord}(f) = \mathbf{s}(f)\,\mathbf{d}(f)\;, \tag{4}$$

where lower case symbols are the Fourier transformed (fft) versions of the respective functions. The $\mathbf{d}(f)$ is the detector frequency response $\mathbf{d}(f) \equiv \text{fft} D(t)$. For simplicity we model $d(f)$ and $D(t)$ as Gaussians having full widths at half max (FWHM) of f_D and T_D. We desire narrow $D(t)$ and broad $d(f)$.

Figure 5a show that the instrument response for the conventional technique is a peak centered at zero frequency, having limited ability to detect higher frequency components. High frequency is often the region of most interest in a typical science signal. (The narrower the rise of a step function the higher the frequency of its components.)

2.2. Heterodyne detection

In contrast to the constant intensity of the conventional method, the heterodyning technique has each of several illumination channel intensities modulated sinusoidally

$$I_n(t) = 1 + \gamma \cos(2\pi t f_M + \phi_n) \,, \tag{5}$$

where γ is the modulation degree and ϕ_n is the phase of each channel. Three or four blurred signals $B_n(t)$ are recorded on the streak camera or detector having multiple channels at phases ϕ_n differing by 120° or 90°. It is useful to employ an alternative designation for the detected channels using the phase in degrees as the subscript, B_0, B_{90}, B_{180} etc. We will ignore γ for now but include its effect in the final result Eq. 12.

The heterodyned detected signal $B_n(t)$ is Eq. 3 modified to include the sinusoidal time dependence of $I(t)$.

$$B_n(t) = [S(t)\, I_n(t)] \otimes D(t) \,. \tag{6}$$

Substitution of Eq. 5 for $I(t)$ forms a sum of the ordinary measured signal plus two beat or moiré terms.

$$\begin{aligned} B_n(t) &= B_{ord}(t) + \tfrac{1}{2}[S(t)e^{i\phi_n}e^{i2\pi t f_M} + \\ & \quad S(t)e^{-i\phi_n}e^{-i2\pi t f_M}] \otimes D(t). \end{aligned} \tag{7}$$

We obtain one stationary, one clockwise, and one counter-clockwise rotating term, versus ϕ.

The beat component is isolated from the ordinary component B_{ord} by taking a set of N phase shifted data and forming a complex linear combination called a "whirl", $\mathbf{W}(t)$, where B_n are numerically anti-rotated to undo each channel's optically imposed phase shift. The whirl in general for N phase channels is

$$\mathbf{W}(t) = \tfrac{1}{N} \sum B_n e^{i\phi_n} \tag{8}$$

For four phase recordings every 90° the whirl is particularly simple:

$$\begin{aligned} \mathbf{W}(t) &= \tfrac{1}{4}(B_0 e^{i0^\circ} + B_{90}e^{i90^\circ} + B_{180}e^{i180^\circ} + B_{270}e^{i270^\circ}) \\ &= \tfrac{1}{4}[(B_0 - B_{180}) + i(B_{90} - B_{270})] \,. \end{aligned} \tag{9}$$

The whirl for 3 phase channels is

$$\begin{aligned} \mathbf{W}(t) &= \tfrac{1}{3}(B_0 e^{i0^\circ} + B_{120}e^{i120^\circ} + B_{240}e^{i240^\circ}) \\ &= \tfrac{1}{6}[(2B_0 - B_{120} - B_{240}) + i\sqrt{3}(B_{120} - B_{240})] \,. \end{aligned} \tag{10}$$

Applying Eq. 8, 9, or 10 to Eq. 7 we get

$$\mathbf{W}(t) = \tfrac{1}{2}[e^{i2\pi t f_M} S(t)] \otimes D(t) \,. \tag{11}$$

because only one beat component survives the sum over many phases. This expression in the frequency domain is

$$\mathbf{w}(f) = \tfrac{1}{2}\gamma\, \mathbf{s}(f + f_M)\, \mathbf{d}(f) \tag{12}$$

where we include the modulation degree (γ) previously taken as unity.

This important equation describes the heterodyning formation of the moiré or beat signal. The heterodyning is the shifting expressed in the $\mathbf{s}(f + f_M)$ argument. Fine temporal details in the science signal $s(f)$ having high frequency f are shifted by f_M to low frequency *prior* to blurring by $\mathbf{d}(f)$. This effectively shifts the sensitivity of the detector to higher frequencies by f_m, as depicted by Fig. 5b.

2.2.1. Extracting the ordinary spectrum

Note that even though the illumination is modulated, the ordinary unmodulated signal is easily obtained from the data by a straight sum of the phase-stepped data (so that oscillatory terms cancel),

$$B_{ord}(t) = \frac{1}{N}\sum B_n \tag{13}$$

or specifically for 3 or 4 phase channels:

$$B_{ord}(t) = \frac{1}{3}(B_0 + B_{120} + B_{240})\ or \tag{14}$$

$$B_{ord}(t) = \frac{1}{4}(B_0 + B_{90} + B_{180} + B_{270})\ . \tag{15}$$

Hence, modulation does not prevent obtaining the conventional signal. The beat signal is new information, obtained without destroying the conventional information.

2.3. Signal Recovery and Response

To recover $S(t)$ at a improved time resolution, the measured beat signal $\mathbf{w}(f)$ is processed to reverse the heterodyning expressed in Eq. 12. This is then combined with the ordinary signal to form a composite output. This is analogous to combining the "bass" and "treble" channels of an audio system to obtain a more full-bodied sound.

The data processing steps include (1) resampling $\mathbf{W}(t)$ to linearize vs t, so that in the next step the modulation comb component forms a very narrow spike in $\mathbf{w}(f)$ at f_M. (2) Fourier transform $\mathbf{W}(t)$ to form $\mathbf{w}(f)$ and translate by f_M toward higher f. Hence low frequency detected moiré components become high frequency signals. (3) Delete the negative branch, which does not hold as significant of signal and is noisier than the positive branch. (4) Inverse Fourier transform and take the real part to form the "treble" signal $B_{het}(t)$. (5) Sum the treble and ordinary signals to form a composite signal, after first weighting the components by an amount proportional to their expected strength vs f. This discriminates against noise for frequencies where the signal is known to be noisy. (6) Equalize the frequency distribution of the composite signal to force the instrument lineshape into a user-desired response such as Gaussian, which minimizes ringing. The weighting function needed for equalization is obtained from a calibration recording on a known signal, which calibrates the detector blurring $d(f)$. Further discussion of this data analysis procedure is in Ref. 3.

3. METHODS OF ENCODING PHASED ILLUMINATION

Figure 7 shows a proposed method of encoding the channels using wavelength. A laser generates a long pulse (sufficiently long to cover the duration of the measurement). This is passed through some nonlinear optical material to generate other wavelengths besides the original laser wavelength. Depending on details of the time scale of the pulse and its intensity, it might be possible to generate a white light pulse, or use 2nd and 3rd harmonic generation. For longer time scales (where the intensity may not be sufficient for nonlinear optics), the single laser could be replaced by 3 lasers operating at different wavelengths. In any case, the multi-wavelength pulse then passes through a device for modulating its intensity. Finally, the phases of the multiple wavelengths are shifted by delaying one wavelength relative to the other. For high f_M, only small delays are needed, and this could be done by the dispersion in ordinary transparent materials such as glass. For lower f_M and thus longer delays, this could be done by separating the wavelengths with dichroic filters or a prism or grating and having the optical path length differ as a function of the wavelength.

4. RELATED DEMONSTRATIONS

Experimental demonstrations of this theoretical technique have not yet been performed in the time domain. However, analogous demonstrations[3–5] have been performed in the *spectral* domain for astrophysics. The software developed for the astrophysical spectral application was used here in the numerical simulation of time resolution boosting shown in Figure 4, by substituting the time variable for the optical frequency of light in the spectral application. In both cases a moire process occurring in the apparatus is reversed numerically during data analysis, that shifts the science signal to lower frequencies in the relevant variable so that it is easier to detect.

The spectral resolution of the Lick Observatory spectrograph was approximately doubled[3] by imprinting a sinusoidal spectral comb on starlight with an interferometer of fixed delay inserted into the beam, and postprocessing the data using equations analogous to those described above. In recent experiments[5] the spectral resolution of a benchtop grating spectrograph measuring the iodine spectrum was boosted approximately 6 times, from its native 25,000 resolution to an effective 140,000, over the full bandwidth of the spectrograph by using multiple heterodyning exposures and combining the individual results. These demonstrations show that reconstruction of a full-bodied data set from multiple down-heterodyned channels is possible. The spectral heterodyning technique has been described by the author in a US patent.[6]

ACKNOWLEDGMENTS

This work was performed under the auspices of the U.S. Department of Energy by the University of California, Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.

REFERENCES

1. L. Barker and R. Hollenbach, "Laser Interferometer for Measuring High Velocities of any Reflecting Surface," *J. Appl. Phys.* **43**, pp. 4669–4675, 1972.
2. W. Hemsing, "Velocity Sensing Interferometer (VISAR) Modification," *Rev. Sci. Instr.* **50**, pp. 73–78, 1979.
3. D. Erskine, J. Edelstein, M. Feuerstein, and B. Welsh, "High Resolution Broadband Spectroscopy using an Externally Dispersed Interferometer," *ApJ* **592**, pp. L103–L106, 2003.
4. D. Erskine and J. Edelstein, "High-resolution Broadband Spectral Interferometry," in *Future EUV/UV and Visible Space Astrophysics Missions and Instrumentation, ed. J. C. Blades, O. H. Siegmund*, pp. 158–169, SPIE Proc. 4854, Feb. 2003.
5. D. Erskine, "Interferometric Resolution Boosting for Spectrographs," in *Ground-based Instrumentation for Astronomy, ed. A. Moorwood*, SPIE Proc. 5492, June 2004.
6. D. Erskine, "Combined Dispersive/Interference Spectroscopy for Producing a Vector Spectrum," *US Patent* **6,351,307**, Feb. 26, 2002.